THE WRITING MATHEMATICIAN

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> *Writing isn't like math; in math, two plus two always equals four no matter what your mood is like. With writing, the way you feel changes everything.*
> —Stephenie Meyer [1]

I feel for my students when I hand them their first essay assignment. Many are mathematicians—students and teachers—who chose to study mathematics partly to avoid writing. But in my mathematics *education* courses, and in the discipline more generally, academic writing is part of our routine practice.

Mathematicians face some challenging stereotypes when it comes to writing. As the Meyer quote suggests, writing is often seen as ephemeral, subjective and context-dependent, whereas mathematics is seen as enduring, universal, and context-free. Writing reflects self, mathematics transcends it: they are essentially unlike each other. This false dichotomy of writing versus mathematics can discourage mathematicians from writing, especially when combined with similarly coarse dichotomies such as *right brain* | *left brain*, *creativity* | *logic*, and *art* | *science*. Taken together, these dichotomies suggest that writing is outside the natural skillset of the mathematician, and that one's mathematics training not only neglects one's development as a writer, but also actively prevents it.

Where does this *writing* | *mathematics* dichotomy come from? It is profoundly unhelpful for our discipline of mathematics education: it assumes a gatekeeping role, turning away potential newcomers who might otherwise have a lot to offer. This essay deconstructs the *writing* | *mathematics* dichotomy by identifying similarities between the practices of academic writing and mathematics. I offer three writing-mathematics metaphors based on these similarities: writing as modelling, writing as problem-solving, and writing as proving (see Table 1). I propose that these metaphors might encourage students who identify more as mathematicians than writers to recognise and replace unproductive writing beliefs and practices with more productive ones that are grounded in familiar mathematical experiences.

I wish to delimit my intentions around this offering. I am not suggesting that mathematicians who read this essay will automatically become highly competent writers, simply by acknowledging similarities between writing and mathematics. Nor am I suggesting that one's skill in mathematical modelling, problem-solving and proving transfer effortlessly to the domain of writing. My goals are more modest and realistic. I want my students to work on their writing, just as they have worked on their mathematics. The three metaphors are offered as encouragement to *begin* this process, rather than being intended as a remedy or solution. I propose that mathematicians do not have to see themselves as starting from nothing when they engage in academic writing. Rather, they can view their writing development as building on competencies they have already honed in their mathematical training, but which they may not have formerly recognised as writerly.

The imperfect ideal: writing as modelling

Let us consider a prototypical mathematics education student who has spent weeks thinking, reading and talking

Writing-mathematics metaphors	Unproductive beliefs and practices addressed	Productive beliefs and practices encouraged
Writing as modelling: The drafting process is like the modelling cycle, where early drafts are created in order to generate improved subsequent drafts	Belief: One should know what to write before one starts writing Practice: Wait until the last minute before beginning to write	Belief: Writing can generate ideas Practice: Create early drafts as a mechanism for figuring out what one wants to write
Writing as problem solving: Writing is like solving a mathematical problem, where getting stuck is natural and expected	Belief: If one knows what to write, the writing should flow easily Practice: Give up when stuck	Belief: Writing can be used to analyse and organise ideas Practice: When stuck, approach one's writing metacognitively, and seek new ways of structuring one's attention to one's ideas
Writing as proving: Academic writing is like a proof that performs a dialogic role in the way it addresses and seeks to convince a public	Belief: Writing is a permanent, inert record of one's knowledge Practice: Dismiss actual and potential reader interpretations of one's writing	Belief: Writing is a dialogue with a public Practice: Seek readers' interpretations of one's writing, from self-as reader, imagined readers and actual readers

Table 1. Metaphor mappings between mathematics and writing practice.

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about her essay topic, but only starts writing it the night before it was due. She writes one draft only—the one she hands in, and is disappointed with the low grade her essay receives. She wishes she had started earlier, but rationalises to herself that she was still trying to figure out what she wanted to say up until the moment she started writing. It was only the pressure of the deadline that forced her to start writing; without it, she would have spent even more time thinking and reading to develop her ideas. After all, she reasons, there is no point writing when you do not know what to write about!

This "think first, write after" approach, sometimes known as the "writing up" model is a dangerous trap that many students fall into, and is at odds with the way writing often works (Menary, 2007). The approach allows no room for an iterative drafting process, whereby imperfect drafts are written that are not intended as final copy, but are necessary steps towards subsequent improved drafts. Writing experts trade on the generative power of imperfect writing—they encourage writers to turn off their internal critic and allow themselves to write badly as a way of overcoming writing inertia and discovering new ideas (Elbow, 1985). Lamott heralds the "shitty first draft" (1994) as an ideal (and achievable!) first goal in the writing process: anyone can produce a sketchy first draft that generates material that can be worked on, improved, and eventually rewritten into a more sharable form.

Mathematical modelling offers a compelling metaphor for the generative power of imperfect writing in the creative process. Like polished writing, polished mathematical models are seldom produced in a first attempt. A modeller typically begins with some understanding of the real [2] situation to be modelled. The modeller mathematises variables and relationships from his or her understanding of the real situation, and 'writes' it into an initial mathematical model. Next, the modeller runs the model to test it, and interprets the results back into the real situation, comparing the mathematical output with real data. At this point, the modeller may notice information in the real situation that was previously missed, create a revised model based on this enhanced interpretation of the situation, and subject the model to further testing and revision, ultimately going through the full modelling cycle multiple times until s/he is satisfied (Borromeo-Ferri, 2006). The model is his or her mathematical description of the situation, written in mathematical notation, and the modeller who publishes a mathematical model has typically created and discarded multiple models along the way, just as the writer who publishes a piece of writing has typically written and discarded multiple drafts along the way.

Novices to modelling might regard this iterative process as a waste of time: Why bother creating models that will only be discarded? The novice might try to bypass the iterations, thinking that if one thinks harder in the first place, one might produce a better model in an initial attempt. But the experienced modeller takes the more efficient approach of entering into modelling cycles, not avoiding them. Models self-propagate: the model one produces to express one's interpretation of a situation becomes a conceptual lens through which one can review the situation (Lesh & Doerr, 2003); on this re-viewing, one can notice deeper levels of structure that can be incorporated into a more powerful model. The modeller who views a real situation through multiple different models will typically notice more than one who spends the same amount of time trying to understand the real situation from an initial, untested point of view in the hope of producing a perfect first model. Similarly, the writer who writes, reads, and revises multiple drafts will likely develop his or her ideas further than the writer who only thinks about his or her ideas in the hope of producing a perfect first draft.

A mathematical model is not valued for its verisimilitude, but for the opportunity it gives the user to manipulate, predict, or explain a system in a way that is not otherwise possible: "All models are wrong. But some are useful" (Box, 1976). Modellers are aware that their mathematical descriptions exaggerate some features of a system and omit others, and they manipulate this consciously to enhance the usefulness of the model. Perfection is simply not a relevant ideal in mathematical modelling; the utility of a model comes from its approximation of reality.

Modelling, like drafting, *expects* imperfection. This can be liberating: one can create a bad first model without worrying that it means one is a bad modeller. Modelling can be a useful metaphor for the generative role of writing for the mathematics education student who does not know what to write for her essay. Instead of waiting to figure out her ideas before writing, she can allow herself to write a bad first draft—full of contradictions, unfinished thoughts, ideas that are not structured well—in order to generate material that can be worked on and improved. Rather than viewing her bad first draft as evidence that she is a bad writer, she can view it as a useful tool for figuring out what she wants to say. She can enter into the drafting cycle knowing that the writing she does now will help her figure out what she wants to say.

The thinking laboratory: writing as problem solving

The generation of ideas is just one of writing's roles; writing also plays a role in the analysis and organisation of ideas that have been generated. Our prototypical student comes close to experiencing this second role when she knows what she wants to write about, but struggles to write it. She has a thesis and outline for her intended essay, but the writing is painstaking. She spends hours writing the first sentence, only to delete it the next day. She throws up her hands and complains that she knows what she wants to say, but does not know how to say it—that she is 'bad at writing', and that if she were 'good at writing' she would be able to convey the same ideas more effectively, more eloquently. Good writers, she thinks, would not get stuck like this; their writing would flow elegantly from their pen.

It is an intimidating expectation. Even the most 'talented' writer can be scared into inaction by the demand for elegant sentences created on the spot: "Be brilliant. Now!" Rather than finding it easy, many writers approach writing as an act of problem solving where getting stuck is a natural and expected part of the process. They may have a clear goal in mind, but do not know how to get there. They write carefully, analysing their writing as they go, shifting their attention back and forth between the writing that has been done and the goal that is beyond their reach. They scrutinise their inscriptions, using their writing as a thinking tool (Ong, 1982) that helps them answer questions like: "Should I start with this point, or leave it until these other two are developed first? Does this sentence move me towards my argument, or does it hint at a weakness that would undermine it? Is my central thesis workable, or should I modify it in light of the arguments in this first section?" Writers may begin with an outline plan, but they are not surprised when they abandon it as their analysis and writing leads them to re-evaluate their goals, and to create and work towards new ones.

Mathematical problem solving has been characterised as involving a similar process of working back and forth between givens and goals and posing new problems while exploring existing ones (Kontorovich, Koichu, Leikin & Berman, 2012). Mathematical problems require the solver to combine known ideas in previously unknown ways to create a solution method that was not already known. Problems have been distinguished from mathematical exercises (Schoenfeld, 1985), which are strings of structurally similar questions that require students to practice a recently learned procedure. A student who has mastered a procedure will complete exercises quickly, easily and accurately, much like the mythical "good writer" who issues a stream of perfect sentences in one sitting without breaking a sweat. This difference is important: exercises require less mental effort because the script is already known; problems are more demanding, as they require the solver to create the script as they go.

Writing an original essay is like trying to solve a problem—there is no script to follow, it must be created by simultaneously determining one's goals and figuring out how to achieve them. In both essay writing and mathematical problem solving, getting stuck is natural and expected—it is even a special kind of thrill. When mathematicians get stuck, they engage in metacognitive activity (Garofalo & Lester, 1985), re-viewing their desired goal and comparing it to their current ways of thinking in order to identify their domain of validity (Brousseau, 2002)—the domain where their ways of thinking work and the point at which they break down. They also try to shift their ways of attending (Mason, 2003) to the problem, often by employing heuristics such as working backwards, drawing a diagram, or solving a related problem (Polya, 1945) in the hope of finding productive ways of thinking. They usually conduct such activities with a writing implement (pencil, chalk, whiteboard marker, keyboard) in hand, creating a "coded system of visible marks […] whereby a writer could determine the exact words that a reader would generate from a given text" (Ong, 1982, p. 83). That is, they write.

This last observation may come as a surprise to mathematicians who do not think of their problem solving activity as writing. Doing mathematics—that is, the ordinary everyday concrete details of manipulating mathematical relationships and objects to notice new levels of structure and pattern—involves scratching out symbols, marks, moving ideas around the page or board (Livingston, 2015). To most mathematicians, writing is synonymous with 'writing up one's results', and the semiotic activity that precedes the 'writing up' is 'working', 'thinking', or 'figuring things out', but not 'writing' [3]. Of course, we are talking about different kinds of writing. The 'writing up' of results is writing-as-reporting, performed at the end of the problem-solving. In contrast, the semiotic activity that precedes 'writing up' is a more analytic form of writing used to restructure one's ways of thinking during the problem-solving process. In academic writing, this writing-as-analysis is often performed without the expectation that it will appear intact in the final draft; it is often performed on scrap pieces of paper, with scribbles and arrows used to help the writer/thinker restructure his or her arguments.

Even if mathematicians do not call such semiotic activity writing, they readily acknowledge that the marks they make on the page, screen or board are fundamental to their mathematical progress—that they help them analyse their ideas, that they "restructure thought" (Ong, 1982). My mathematician colleague Rod Gover once declared, "The whiteboard is our laboratory", when arguing for more space in university buildings. Another mathematician, Steven Galbraith, told my class, "I use a whiteboard when the ideas are too big for my head", conjuring an image of the whiteboard as an extension of the mind, with thinking distributed across multiple modalities. These marks help separate the known from the knower (Ong, 1982), a distancing that allows the mathematician to "examine [one's] thoughts more consciously as a string of assertions in space" (Elbow, 1985, p. 284) than if one spoke or thought them without writing them. Such distancing facilitates abstraction, which is a mathematical ideal.

Then why not call it writing? I suspect the reluctance stems from a disciplinary tendency to overestimate the role of purely mental faculties and underestimate the role of the diagrams, symbols, gestures and glances that help mathematicians see new levels of structure. The archetypal story of mathematical discovery tells how Poincaré unexpectedly realised the link between non-Euclidean geometry and complex function theory while stepping on a bus, resumed his previous (non-mathematical) conversation for the rest of the bus ride with quiet certitude of his discovery, and only verified it when he got home—presumably with pen and paper (Hadamard, 1945). The story creates a powerful image of pure thought (or divine inspiration) as the source of mathematical discovery, and diminishes the role of writing to verification. Mathematicians' reluctance to call their analytical writing 'writing' may also be motivated by a mathematical aesthetic that depends on the purity, simplicity, and abstraction of mathematics from the materiality of the so-called real world: ink and chalk dust are substantial, but the ideas they express endure and transcend. Perhaps this aesthetic prevents some mathematicians from acknowledging their role as writers who deal in messy physical marks, preferring instead to consider themselves thinkers reflecting on the ideal forms those marks express.

Why do I care that mathematicians acknowledge their semiotic activity as 'writing'? Quite frankly, because they are good at it. They have spent years honing their ability to use writing to restructure their thoughts, to dissect their ideas, identify new arguments—they possess an analytic discipline that most writers struggle with. Yet few of my mathematics education students take advantage of this in their academic writing. They want their writing to come out in consecutive, polished sentences, and become discouraged when it does not, rather than using their writing to analyse and probe their arguments as they do when they are stuck on mathematical problems. By viewing writing only as a medium for communicating perfectly formed thoughts, they deny themselves their own laboratory, their own thinking tools.

I am not suggesting that one's success in solving mathematical problems automatically translates into successful essay writing. But the metaphor of writing as problem solving may encourage a mathematics education student not to give up too easily when she finds herself stuck in her writing. Perhaps she can approach her writing metacognitively, reviewing her central thesis in light of her arguments, articulating her goals and identifying limitations in her current ways of thinking like she does when working on a mathematical problem. She may even try using problem-solving heuristics to break her current ways of attending to her writing, and attend to her writing (and ideas) in new ways. At the very least, she may come to view her stuckness as a natural and expected state (Burton, 2004), just as mathematicians do with mathematics, and recognise it as an opportunity to analyse and restructure her ideas.

Entering into dialogue: writing as proving

Our prototypical student now has a good draft that she is happy with. She is satisfied it represents her knowledge of the subject matter, and has read extensively to check the accuracy of its content. A friend reads the draft and finds it difficult to understand. Unperturbed, our prototypical student attributes the reader's difficulty to insufficient subject knowledge. She is confident that her essay demonstrates her mastery of the topic, and that this will be recognised by her more knowledgeable teacher.

But the essay is not merely an inert record of a writer's knowledge, and its quality is not merely judged on the number of correct facts it contains. The essay is also a rhetorical act that seeks to engage a public. It has to do real work as a dialogic tool: it must address an audience; it must convince or persuade a public; it must inspire some kind of response or action. Writers take considerable care to shape their writing's addressivity (Bakhtin, 1986) by seeking insight into their readers' experience of the writing.

Mathematical proofs are like expository essays in this regard: they must also convince an audience. When an undergraduate mathematics student switches from memorising other peoples' proofs to constructing proofs of her own, she becomes concerned with how her proof (writing) performs under dialogic conditions. She is encouraged to test her proofs on different audiences: "convince yourself, convince a friend, convince an enemy" (Mason, Burton & Stacey, 2010). She may take an undergraduate course in proof construction, in which she learns how to read and evaluate proofs as part of learning how to write and construct one (Weber, 2010).

Mathematicians who construct proofs are like writers who create utterances as part of a dialogic chain in order to move, to persuade, "to do things to people" (Elbow, 1985, p. 300). They are conscious of their audiences, and will often vary the style, amount of detail and linguistic features of a proof (Pimm & Sinclair, 2009), depending on whether it is being presented in real time to close colleagues, written in lecture notes for a graduate class, or submitted to a research journal. Moreover, these different modes of address can sometimes be generative: the awareness of different audiences' needs may alert the mathematician to new mathematical details or big picture isomorphisms that need to be worked out.

Mathematicians actively seek out listeners and readers who can identify weaknesses in the proofs they are constructing. Perhaps our prototypical student could enhance her writing by similarly seeking ways to evaluate her writing's dialogic performance. She could read her writing aloud to 'hear it'; she could ask someone else to read it and tell her where their attention wanders; she could imagine herself writing for a particular person or audience; she could leave the essay and read it at a later date through the eyes of a reader. Such practices can result in productive tensions, where the reader sees something different to what the writer intended: "That's not what I meant!" And the writer reenters into dialogue, revising her argument, with a deeper understanding of what she is trying to say.

Reflections

I have challenged the common perception that writing is opposite to mathematics by offering three metaphors that highlight similarities between aspects of writing and mathematics practices (see Table 1). According to Sword (2017), "metaphors, after all, are the stories that we tell ourselves about our relationship to the world. By changing our metaphors, we can rewrite our stories" (p. 191). I hope these metaphors will help students who identify more as mathematicians than writers to re-story themselves as writing mathematicians, who use writing as a tool for thinking more deeply about the mathematics and mathematics-education questions they care about most. I offer these metaphors with the conviction that it is ethically and fundamentally imperative that we support our mathematics education students' writing development: not only to help students reach their academic potential, but to ensure our conversations include diverse voices (Geiger & Straesser, 2015) that are critical to our discipline's progress.

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Notes

[1] http://stepheniemeyer.com/2008/08/midnight-sun

[2] Readers may object to the use of the term 'real'—is mathematics not real? Some writers on modelling us the term 'extra-mathematical' instead of 'real' to acknowledge the ontological slipstream; Borromeo-Ferri (2006) points out that some demarcation between mathematical and 'real' or 'extra-mathematical' worlds is necessary and the use of 'real' to describe the 'extra-mathematical' can be a pragmatic choice.

[3] See also Latour and Woolgar's (1979) anthropological observation of laboratory scientists as "compulsive and manic writers […] who spend the greatest part of their day coding, marking, altering, correcting, reading, and writing" (pp. 48-49).

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